THE VEE FORMATION OF CANADA GEESE

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THE organized vee formations of geese and other waterfowl have drawn attention from sportsmen and scientists for years. Lissaman and Shollenberger (1970) proposed that birds may fly in a vee to capitalize on upward rising components of the wingtip vortex currents generated by the wings of neighboring birds flying to the front and side. Franzisket (1951) felt that tip vortex energy was not important in formation flight, and Cone (1968) suggested that the tip vortex flowing from a flapping wing is quite different from that arising from the fixed wing of aircraft. Determination of the geometry of vee formations is necessary for resolution of the question of possible aerodynamic advantage in vee formation flight. We report here the first determination of the angle between the legs of vee formations of Canada Geese (*Branta canadensis*) and measurements of the distances between birds in formation.

METHODS

Determination of vee angle.—We filmed migrating flocks of Canada Geese at the Montezuma National Wildlife Refuge, Seneca Falls, New York, during the week of 5–9 October 1971. The vantage point from which the birds were filmed was a dike between two ponds. The camera position was exposed, and the birds avoided flying directly overhead.

Films were taken with a Beaulieu 4008ZM Super 8 cine camera at 18 frames/second. The extension of the 8–64 mm lens depended on the distance from the camera to the flock being filmed. In most cases, a 64 mm focal length was necessary, but some flocks flew close enough to the camera position to require a shorter focal length to keep the entire flock in the viewfinder.

Filming of a particular flock usually started when it was heading toward the camera, and when the individual birds were visible in the viewfinder. Filming stopped when individual birds could no longer be resolved.

The camera was mounted on a tripod that had been modified to act as a maximum recording inclinometer, permitting measurement of the maximum inclination of the optical axis of the camera above the horizon ($\pm$ 0.5°) as the camera tracked the flock along its flight path. This measurement was essential to calculate the true angle between the legs of the formation. The tripod was leveled with a bubble level at each filming session, to provide the horizon reference.

Relationship between apparent and true angle.—The apparent angle between the legs of a vee changes as the birds move along their flight path, and the relationship between the apparent angle seen by the observer and the true angle depends on the height of the birds above the ground, and on the distance from the observer to the flock and flight path. For the following discussion, the reader is urged to draw a 30° vee on a piece of stiff cardboard, and manipulate it himself to visualize the effects reported here.

Figure 1 demonstrates the change in apparent angle of the vee as the flock moves along a straight flight path. At the point of closest approach of the flock
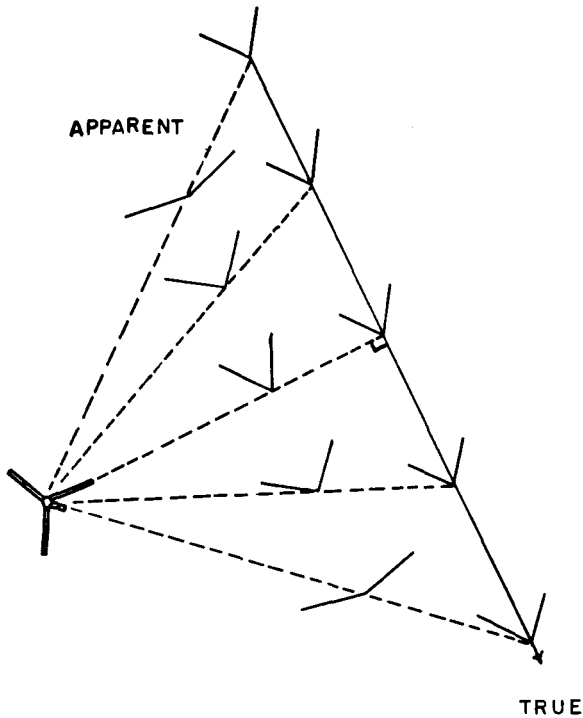
Figure 1. Overhead view of a vee formation flight path passing by a camera on a tripod.

to the observer, i.e. that point where a line drawn between the observer and the leader of the flock would form a right angle with the flight path, the apparent angle of the flock is at a minimum. As the flock moves away from the observer, the apparent angle becomes greater. This relationship was determined empirically by drawing a vee on a board mounted on a moveable dolly, and measuring the angle of the image of the vee in a camera viewfinder as the dolly was pushed by the camera, thus simulating the appearance of a vee formation of birds, all in the same level plane, flying on a straight flight path past a camera position on the ground. The reader can confirm this effect by sitting in a low chair, and having someone walk by him, holding a cardboard vee above his head. It will also be noticed that as the vee approaches, the leg of the vee farthest from the observer will appear to be shorter than the other, even if both are actually of equal length, and as the vee passes away from the observer, the nearer leg will appear to be shorter. The magnitude of this illusion will depend on the distance of the flight path from the observer. If the straight flight path passes directly over the observer's head, both legs will appear to be equal during the whole flight, but the apparent angle will still reach a minimum when the vee is closest to the observer, in this case, when the vee is directly overhead.

If a photographer were following the flight of a vee of geese on a straight flight path, as the birds flew from horizon to horizon he would be forced to move
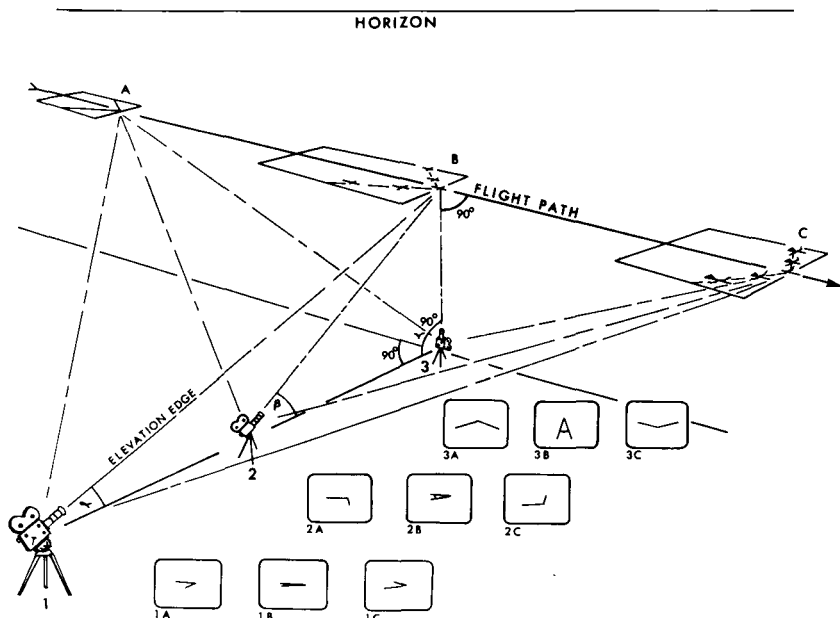
Figure 2. View of a flock flying by camera located at three different positions. For explanation see text.

his camera as the formation progressed. As he did so, if he attempted to keep the flock centered in his viewfinder the front of his lens would describe an arc. The apogee of the arc would be reached as soon as the birds reached a point on their flight path where they were closest to the photographer's position. This point is also the one where the apparent angle of the formation reaches a minimum. Thus, if a series of pictures of the formation were taken as it flew by, the picture that showed the apparent angle of the formation at a minimum would have been taken when the camera was pointed highest above the horizon. This relationship permits the determination of the true angle, as will be described below.

The distance of the observer from the flight path of the birds determines the relationship between the minimum apparent angle of the formation as it flies by, and its true angle. Figure 2 shows the effects of distance of the observer from the formation. Three assumptions are inherent in the following discussion: (1) that the birds are all flying in a plane parallel to the ground, (2) that the flight path is a straight line, and (3) that all members of the flock are flying at the same airspeed, i.e. the true angle is not changing.

The small boxed figures represent what would be seen in a camera viewfinder at camera positions 1, 2, 3, while the formation moves from location A to B to C. At position 3, a location directly under the flight path, the minimum apparent angle equals the true angle. At this point, the birds are directly overhead. From this camera position, legs of equal length appear to be equal in the whole progress of the flight, but the angle between the legs approaches 180° as the birds leave

or reach the horizon. At positions 1 and 2, which are successively more distant from the flight path, the minimum apparent angle becomes less than the true angle, and if the flight path were on the horizon, relative to the camera, the angle between the legs of the vee would approach 0°. The reader is again invited to test these observations by manipulating his cardboard vee.

Summarizing the relationship so far, we find that as a camera tracks a vee along a straight, constant altitude flight path, the apparent angle of the formation reaches a minimum as the vertical angle of the camera above the horizon reaches a maximum. As the camera position moves away from a point directly under the flight path, the minimum apparent angle decreases. The relationship between the minimum apparent angle and the true angle is a function of the angle of the camera above the horizon (angles α, β, γ in Figure 2). If the angle of the camera above the horizon is known at the point of minimum apparent angle, projective geometry can be used to convert the apparent angle to the true angle.

DATA ANALYSIS

A Kodak MFS–8 motion analysis projector was used for examination of film, which was projected on a 20 $\times$ 27.5 cm screen. The films were first examined to determine types of formations and the number of birds in each formation. Angular formations were further analyzed if they met the following requirements: (1) the formation was clearly visible as a group of individual birds, not just as shapes silhouetted against the sky, (2) the formation persisted throughout most of the take, i.e., the birds maintained their positions relative to one another within the flock, so the shape or type of the formation did not change during the take.

Those flocks that met these conditions were then studied to obtain the angle between the legs of the formation, the wingbeat frequency of each bird, wingbeat phase relationships among the birds, and the distance between adjacent birds along the legs of a formation (where possible).

To obtain the true vee angle of a formation, it was first necessary to determine which frame of the take represented the minimum apparent angle. As the minimum angle was not immediately obvious upon scanning the film, it was necessary to determine the apparent vee angle of the formation in a series of frames; the smallest angle in this series represented the minimum angle. Each frame in a series within a particular take was projected on a graph paper screen and the images of the birds in the formation traced onto the paper. X and Y coordinates were assigned each bird in a frame, the "center" of each bird being used for the coordinate point. Because of limitations in resolution of the Super 8 system, and changing perspective as the formation flew by, this "center" point was the only location on the birds' bodies that could be used consistently for all formations. The "center" was visually estimated as being the center of mass of each bird, below the wings. The image of each bird was numbered, the lead bird as No. 1, the bird immediately behind No. 1, and on the observer's side of the formation, as No. 2, etc. Those birds in the leg of the formation that appeared farther from the observer were numbered No. 2', No. 3', etc., following the convention of Nachtigall (1970). After assigning coordinates to each bird in a formation, a linear regression line was calculated for each leg of the formation. When the regression lines were completed, the angle formed by their intersection was measured.
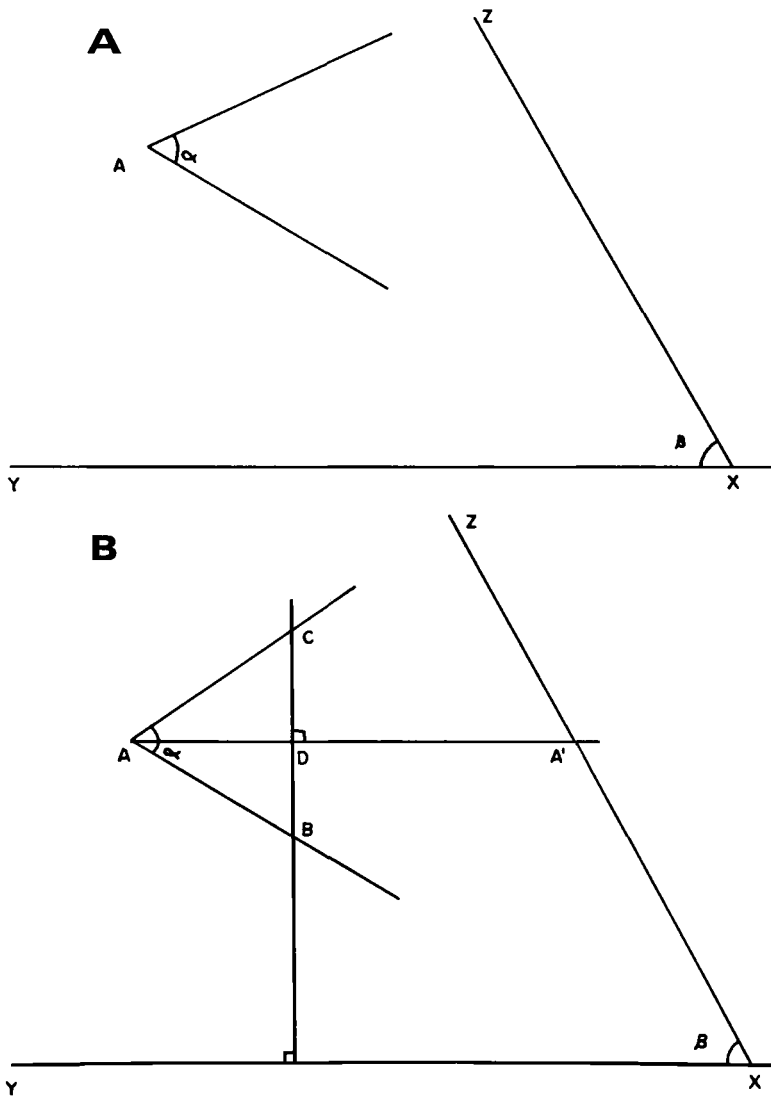
Figure 3. A–D, projection of an apparent vee angle to a true one, and determination of distances between birds. For explanation see text.

The minimum apparent angle between the legs of the formation, obtained from the regression lines, was converted to the true vee angle of the formation by using projective geometry (three-dimensional descriptive geometry). Figure 3 (A-D) illustrates how the minimum apparent angle is projected upon the camera elevation angle, then projected as the true angle of the formation (Slaby 1966).
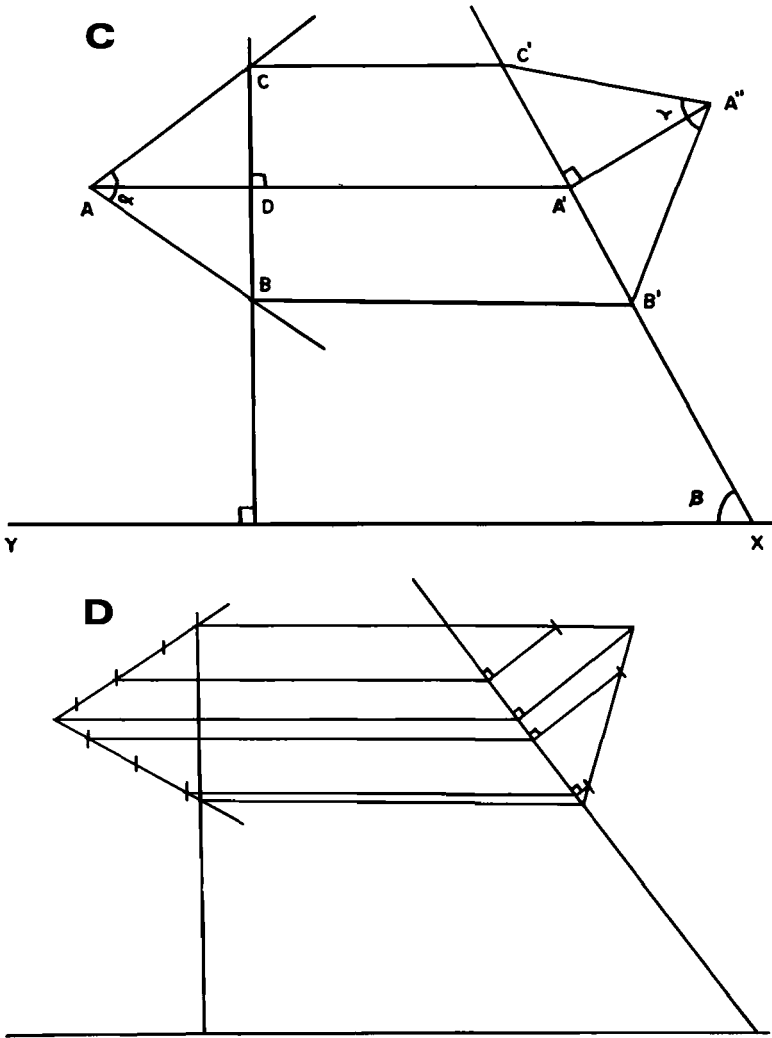
Figure 3A shows the minimum apparent angle of a formation. Line XY is the ground plane the camera is standing on, A is the apex of the formation, angle α is the apparent angle, and angle β is the elevation of the camera above the horizon when the picture was taken. The distance from A to XY is irrelevant, as the method is not intended to calculate the height of the formation above the ground. To project angle α so the true shape and angle of the formation can be determined, a line is drawn from point A parallel to line XY, and intersecting XZ at point A'. Figure 3B shows the position of line AA' relative to the elevation edge (XZ). To perform the projection of the angle, at least three points must be projected onto the elevation edge (line XZ). Point A' has already been projected onto the elevation edge; two more points are needed. In Figure 3B, a

line is drawn perpendicular to AA' and XY, through AA' and intersecting both legs of the filmed formation. The distance of this line from the apex of the vee (point A) is irrelevant as long as the line remains perpendicular to AA' and intersects both vee legs. The points at which this line intersects the vee leg nearest line XY, intersects line AA', and intersects the vee leg farthest from line XY are labelled B, D, and C respectively. In Figure 3C a line is extended from point B to line XZ, perpendicular to line BC, and from point C to XZ, perpendicular to BC. We now have points A', B', and C' on line XZ, the elevation edge. A line is now drawn perpendicular to XZ at point A'. The line A'A'', is equal to line AD. Lines are then drawn from B' and C' to A''. The angle C'A''B' (angle γ) is the true angle of the formation.

Several sources of potential measurement error can be identified. If the camera elevation angle is not accurate, the calculated true angle will be in error. The magnitude of this error is a function of the maximum vertical angle. A 2° error in vertical angle measurement when the camera is at a 45° vertical angle will produce a 1.4% error in the calculated true angle. A 2° error in vertical angle measurement when the camera is at a 20° vertical angle will produce a 5.3% error in the calculated true vee angle. The smaller the apparent angle of the vee, the greater will be the error in the calculated true angle, as the thickness of the lines drawn in the projection becomes a factor.

Distance between birds.—An addition to the basic projective geometry technique was used to determine the true distances between adjacent birds along the legs of a formation. The distance between the projected images of two adjacent birds was measured, from the center of one bird to the center of the next, and as if the birds were lying directly on the regression line (birds whose center did not lie on the regression line were connected to it by extending a line parallel to the Y-axis from the center of the bird to the regression line. The point of intersection with the regression line was used to represent the center of the bird). The formation was then drawn on graph paper, and the points representing each individual bird projected onto the elevation edge and the true angle (Figure 3D). The new distances between the birds were measured, and used in the following equation to determine the true distance between the birds:

$$X = \frac{(\text{apparent projected distance}) \quad (\text{true length})}{(\text{apparent length})}$$

where:

X = the true distance between two adjacent birds

Apparent projected distance = distance measured on the projected formation.

True length = the means of bill-to-tail length of specimens of *Branta c. canadensis*, obtained from data in Ruthven and Zimmerman (1965), Terres (1968), and the fieldwork of George Bond (MS). This value was 85.2 cm and was used as a constant for all formation studies in this project.

Apparent length = the average of the bill-to-tail length, for the birds in a particular leg of a formation, as they appeared in the film frame depicting the minimum apparent angle.

For example in formation B (Tables 2–4) the average bill-to-tail length of the birds on the film frame of the minimum apparent angle was 0.81 cm. The apparent projected distance between bird No. 4 and bird No. 5 was 4 cm.

Substituting these values into the above equation, one obtains:

TABLE 1

TYPES OF FORMATIONS AND NUMBERS OF BIRDS IN EACH FORMATION TYPE

Formation type[1]	Number of formation type counted	% of total	Mean number of birds per flock[2]
Column/echelon	41	39.4	21.8
Vee	17	16.4	21.6
Jay	16	15.4	27.9
Compound vee	6	5.8	60.0
Cluster	18	17.3	17.9
Inverted jay	1	0.96	(22)[3]
Front	4	3.8	21.8
Vee with birds inside	1	0.96	(25)[3]
TOTAL	104 flocks	100	$X = 27.2$

[1] Description of formation types in Heppner and Haffner (1974).
[2] Not including flocks with more than 100 birds.
[3] Number in single flock observed.

$$X = \frac{(4 \text{ cm}) \quad (85.2 \text{ cm})}{(0.81 \text{ cm})}$$

therefore:

$$X = 420 \text{ cm} = 4.2 \text{ m.}$$

Wingbeat frequency.—Wingbeat frequency was analyzed by marking the wing position of each bird in a formation for several frames to determine the number of frames for completion of wingbeat, and hence beats/second. Four wing positions were described and assigned letters as follows: A = maximum extension of the wings during the upstroke; B = maximum bending of wings during the upstroke; C = extension of the wings on a horizontal plane, during the downstroke; D = maximum extension of the wings on a vertical plane during the downstroke.

Therefore, a sequence depicting one complete wingbeat would read B, A, C, D, B, etc. Each bird in the frame was given a letter representing its wing position. Approximately 20 frames per formation were analyzed, and an average wingbeat frequency for each bird calculated. Phase relationships were studied from the same data.

RESULTS

Of the 34 linear formations filmed, 5 met the criteria for further analysis. The majority of the flocks were disqualified because of lack of persistence of the formation, i.e. the birds within the formation changed positions frequently, so that the angles were also changing. The true angle was determined for the 5 formations, but of these 5, only 3 could be used to determine the distances between adjacent birds; problems of perspective and distance of the flock from the camera made two of the flocks unusable. Table 1 lists the types of formations filmed and the average number of birds in each formation type.

Table 2 shows the true angles, determined by projective geometry, of the five formations. The minimum apparent angles and camera eleva-

TABLE 2

TRUE ANGLE, MINIMUM APPARENT ANGLE, NUMBER OF BIRDS PER
FLOCK, AND VERTICAL ANGLE, OF FORMATIONS

Formation	Number of birds	Minimum apparent angle	Camera vertical angle	True angle
A	14	18.5°	32.0°	27.5°
B	12	24.0°	52.0°	29.5°
C	9	22.5°	39.5°	35.25°
D	15	5.75°	7.0°	44.0°
E	39	15.5°	20.5°	34.75°
Mean ± SD	17.8 ± 12.1			34.2° ± 6.40°

tion angles are given also, to demonstrate how deceptive the apparent angle can be when viewed from different perspectives.

Table 3 shows the true distance between adjacent birds along the legs of a formation, computed as described below. The overall mean distance between birds was 4.1 m (SD = 0.79); the distance ranged from 2.5 m to 12.8 m.

Table 4 shows the wingbeat frequencies for each bird in formations B, C, and D. The frequencies differed very slightly among the formations. The overall mean frequency was 4 beats/second. There was no

TABLE 3

DISTANCES BETWEEN ADJACENT BIRDS ALONG LEGS OF FORMATION

Formation A True Angle: 27.50°		Formation B True Angle: 29.50°		Formation C True Angle: 35.25°	
Bird No.	Distance (m)	Bird No.	Distance (m)	Bird No.	Distance (m)
1–2	3.4	1–2	7.5	2–3	3.2
2–3	12.8	2–3	3.8	3–4	2.8
3–4	5.2	3–4	3.6	4–5	2.8
4–5	6.7	4–5	4.2	Mean	2.9
5–6	2.8	5–6	3.1	1–2	3.9
6–7	2.6	6–7	3.6	2–3'	3.6
7–8	3.1	Mean	4.3	3'–4'	3.9
8–9	3.1	1–2'	5.2	4'–5'	2.5
9–10	2.6	2'–3'	4.0	5'–6'	2.8
Mean	4.7	3'–4'	5.2	Mean	3.3
2–3'	4.2	4'–5'	8.3		
3'–4'	2.7	5'–6'	3.1		
4'–5'	3.3	Mean	5.0		
5'–6'	4.4				
Mean	3.6				
Mean for formation	4.4 ± 2.8	Mean for formation	4.7 ± 1.7	Mean for formation	3.2 ± 0.6
GRAND MEAN 4.1 ± 0.8					

TABLE 4

WINGBEAT FREQUENCIES

Formation B True Angle: 29.50°		Formation C True Angle: 35.25°		Formation D True Angle: 44.0°	
Bird No.	Wingbeats/min.	Bird No.	Wingbeats/min.	Bird No.	Wingbeats/min.
1	227.4	1	270.0	1	216.0
2	227.4	2	245.4	2	231.0
3	240.0	3	245.4	3	231.0
4	240.0	4	225.0	4	231.0
5	240.0	5	270.0	5	231.0
6	240.0	Mean	251.2	6	231.0
7	240.0	3′	254.4	7	231.0
Mean	236.4	4′	216.0	8	231.0
2′	240.0	5′	216.0	Mean	229.1
3′	227.4	6′	245.4	2′	216.0
4′	227.4	Mean	232.9	3′	231.0
5′	254.4			4′	231.0
6′	227.4			5′	231.0
Mean	235.4			6′	231.0
				7′	249.6
				8′	231.0
				9′	249.6
				Mean	233.8
Formation Mean	235.9 ± 8.6	Formation Mean	243.0 ± 21.4	Formation Mean	231.5 ± 8.7

GRAND MEAN 237 wingbeats/min. (SD = 5.8) or 4 wingbeats/sec. (SD = 0.2)

evidence for phase relationships among the birds in a leg of any formations studied.

Independent test of results.—On two separate occasions, fortuitous 35 mm pictures were taken of birds in vee formation as they flew directly overhead, where the true angle equals the apparent angle. On 25 November 1972, a symmetrical vee of 20 Canada Geese was photographed overhead at Bombay Hook Wildlife Refuge, Delaware. The true vee angle was 36°. In October 1972, a 24-bird formation of Great Cormorants (*Phalacrocorax carbo*) was filmed over Narragansett Bay, Rhode Island. The true angle of the vee was 31°. Both of these direct measurements fall within the range of the indirect measurements reported above.

DISCUSSION AND CONCLUSIONS

The formation angles analyzed in this study ranged from 27.5° to 44.0° and were far more acute than previous hypothetical models had predicted (Poncy 1941, Lissaman and Shollenberger 1970). Frequent position shifts within formations prevented determination of the vee angle in a majority of the flocks filmed; flocks rarely maintained a rigid structure.

Distances between adjacent birds along the legs of a vee were varied, and in several cases appeared longer than would seem feasible if the birds were to gain lift from adjacent vortex currents. If the tip vortices are strong, however, then the possibility of lift gain, even at seemingly longer distances, does exist. (The long distances shown in this study could be an artifact of the technique used to analyze the formations. This technique was based on the assumption that the members of a formation were in the same plane; if the birds were in different planes, our values of distances probably would be incorrect by a few meters. Terrestrial and aerial observations of linear goose formations demonstrate that planar differences are probably not great.)

The wingbeat synchrony previously proposed by Geyr von Schweppenburg (1952) and Nachtigall (1970) did not appear in this study. Nachtigall suggested that wingbeat phase relationships were necessary for formation-flying birds to utilize each other's tip vortices; his motion picture analyses of goose flocks demonstrated phase relationships. Our data suggest the birds' wings are acting as independent oscillators of slightly varying frequencies; sometimes some birds will appear to be in phase, but these relationships will not persist unless the frequencies are identical. Berger (1972) also did not discover phase relationships in the wingbeats of formation flying geese. Lissaman and Shollenberger (1970) suggest that phase relationships are not necessary for the utilization of vortex energy.

Charles Blake (pers. comm.) has pointed out to us that in calculating the aerodynamic advantage of formation flight for fixed wing aircraft, the critical measurement is not the distance between the "centers" of the aircraft, but the distance along the flight path between the tips of the wings. At the vee angles and distances between bird "centers" we report, the difference between center to center distance and tip to tip distance is small, approximately 5%. Thus our measurements should be useful once more is known about flapping wing aerodynamics.

As seen in Table 1, the vee formation appears less frequently than other formation types, especially the echelon. Lissaman and Shollenberger hypothesized that only a vee or jay formation could give equal drag distribution among the flock members.

As over 40% of the linear formations filmed in this study were echelons or fronts, i.e. formations with no apex, it is necessary to consider either that the lead bird does not need birds on either side of it for favorable upwash conditions, or that there are nonaerodynamic reasons for formation flight. Forbush (1912) and Bent (1925) suggested that linear formations enable each bird (excluding the leader) to see the other flock members and maintain a clear field of vision to the front. The angle of a linear formation could be related to the position

of the eyes within the head (Heppner MS). Thus, the shape and angle of a linear formation could be functions of the air space requirements and visual needs of the birds. The use of formations could be to maintain flock unity, perhaps aiding in migratory navigation.

Future investigations should resolve many questions. Studies on other formation-flying species are needed, measuring not only angles and distances between birds, but also the conditions (such as weather, wind, altitude, flock number, season, etc.) under which various formations are used. Wind-tunnel studies will be of particular interest, for they will reveal the airflow conditions around birds in flapping flight, and allow comparative studies of birds of different sizes, and birds in solitary flight versus those in formation. Such studies would determine the shape and strength of the tip vortex, revealing if this vortex is usable as a lifting force. This research could lead to a sounder understanding of formation flight, and of why linear formation flight seems to be the preserve of relatively large birds.

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Note added in press (27 November 1973).—Timothy Williams of State
University of New York-Buffalo has just informed us that he and two
of his students, Thomas Klonowski and Phillip Berkeley, have obtained
data on the angle between the legs of vees of Canada Geese using radar
techniques. The mean angle of his formations was $71.53°$, SD $22.8°$ (n
$= 54$ flocks). The angles of the flocks we measured fell within the range
of Williams' sample, but were clustered at the acute end of the range.
Williams' studies were made in spring while ours were made in fall;
both involved flocks of birds flying from roosting to feeding areas, rather
than making long migratory flights.

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